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Quaternary climatic fluctuations as a consequence of self-organized criticality

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It has long been known that the earth has gone through several colder climatic periods – the so-called ice ages – during the late Quaternary. In spite of various hypothesis about the cause of the growth and decay of the ice sheets, there is still no quantitative explanation for the full range of climatic variation. In this paper it is demonstrated that the concept of self-organized criticality due to Bak et al. (1987, 1988) offers a simple and appealing possibility to explain the power law background spectrum of the Quaternary ice volume fluctuations. We present a simple cellular automaton which is able to reproduce almost the whole spectrum of Quaternary glaciation variability. The surprising confirmity of the very coarse model and the data suggests that the continental ice sheets – like the model – are in the self-organized critical state. This is confirmed by results obtained with a global ice model.

At the end of the last century the idea of former extended glaciations became widely accepted [1]. Soon after their discovery the ice ages were related to variations of the earth's orbital elements [2]. The most successful theory is due to the Yugoslavian mathematician Milutin Milankowič, who claimed the variation of the solar summer insolation at high northern latitudes [3,4] was sufficient to explain the glaciation cycles.

The orbital theory was tested with continuous climatic records obtained from marine sedimentary cores [5]. The ice volume record derived from the drilling core Meteor 13519 [6] is shown in fig. 1.

It has been demonstrated that the spectrum of the ice volume variations contains peaks at the astronomical forcing periods of 19, 23 and 41 ky (1 ky = 1000 years) [7], in accordance with the Milankovič theory. However the main portion of the variance of the $\delta^{18}O$ record has a period around 100 ky. This coincides with the astronomical period of the orbit eccentricity, but is associated with negligible insolation forcing. Therefore models which are based on the assumption of a linear transfer can only reproduce a small fraction of the data variance [8–10]. Of course some non-linear process could shift energy from the 19 and 23 ky bands to the difference frequency, which corresponds

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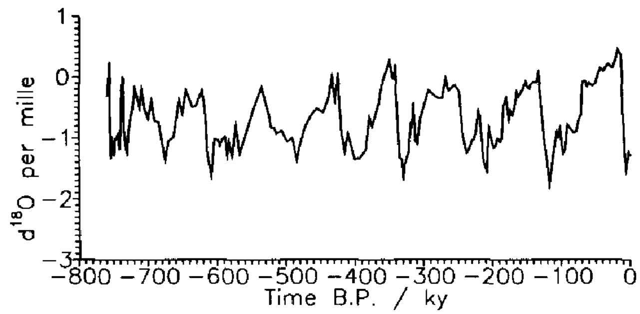

Fig. 1. The Quaternary fluctuations of the oxygen isotope abundance ratio in the ocean, which is a measure of the global ice volume, derived from the marine sedimentary core Meteor 13519. Dating due to the SPECMAP time scale [20].

just to a period of 100 ky, but the question remains how the power at 100 ky can be so much larger than the power at 19/23 ky.

Many extended systems with two or more dimensions and non-linear diffusion dynamics tend to achieve and maintain a self-organized critical state [11–13]. A widely used paradigm for such a system is a sandpile on a limited surface where sand can fall off the edge [13,14]. If the sandpile is built from scratch by slowly adding sand, it at first grows steadily, with the typical size of the largest occurring avalanches increasing with the size of the pile. When a certain size is reached, the increment is on average balanced by sand falling off, and the small perturbations of the added sand can yield reactions of any size, from the movement of a single grain up to an avalanche of the size of the whole system. In this *self-organized critical state* [14,15], there is no characteristic length scale and consequently no characteristic time scale. Hence physically observable quantities obey power laws, cut off by finite-size-scaling effects. The global features of the system do not depend on details of the microscopic dynamics.

To simulate the behaviour of a system like the continental ice sheets in the self-organized critical state, we introduce a cellular automaton which can be taken as toy model of an ice sheet with *ice-surge* dynamics. Ice surging has been observed in glaciers and it has been suggested to occur also in large ice sheets [16–18]. A glacier surge is a short period (between a few months and a few years) of rapid sliding with a velocity up to 100 times larger than the normal ice flow due to deformation. There is no complete theory for the triggering of a surge, but it is generally accepted that the cause is an instability in the glacier itself and that surging requires the presence of basal water.

In our model we neglect normal ice flow and make the following simplifying assumption for the triggering of a surge: When the *thickness* of an ice sheet is greater than a certain value at a particular location, the geothermal flux causes ice melting at the ground, and surging occurs. To simulate this behaviour we consider a rectangular grid of ice columns with heights $h(x, y)_{x,y=1,\dots,N}$. The addition of one ice unit at a particular location increases the height at that location by one. If the height (the "ice thickness") at (x, y) becomes larger than a critical value h_c, the ice can "slide away", which is expressed by

$$h_t(x, y) = h_{t-1}(x, y) - 4\,, \quad h_t(x \pm 1, y) = h_{t-1}(x \pm 1, y) + 1\,,$$
$$h_t(x, y \pm 1) = h_{t-1}(x, y \pm 1) + 1\,. \tag{1}$$

This is exactly the diffusion equation which was used by Bak et al. [14,15] to describe the dissipation of *slope* in a sandpile. In our ice surge model it is equivalent to the behaviour of the *height* itself. The boundary conditions $h(x,0) = h(0, y) = h(x, N+1) = h(N+1, y) = 0$ allow the ice to leave the system (to "melt") at all four edges.

The model is driven by the repeated addition of one unit of ice at a randomly selected position (x, y). After each addition all possible avalanche adjustments are carried out before the process is continued with the next ice unit, i.e. it is assumed that the time scale of the avalanches is small compared with the time scale of the ice build up.

The power spectrum of the resultant time series of the total amount of ice obeys a power law with an exponent close to -2.0, see fig. 2. The same power law spectrum was found for the total amount of sand in the experimental investigation of a real sandpile [13]. The only free parameter of the model – the size N of the system – determines the cutoff point of the power law spectrum by a finite-size scaling effect (together with the trivial scaling due to the time interval Δt attached to the duration between ice additions). The spectrum presented was obtained with $\Delta t = 1$ ky and $N = 12$.

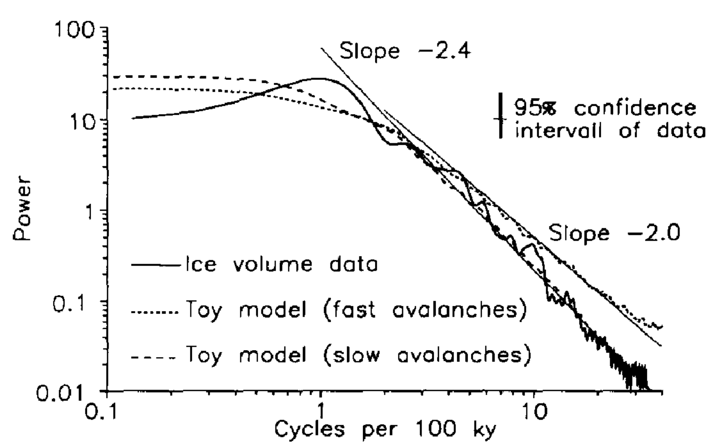

Fig. 2. Power spectra of the Quaternary ice volume fluctuations and the volume fluctuations in the toy model for "infinite fast" avalanches and an avalanche time scale comparable to the ice build-up time scale.

We now slightly change the driving of the model by dropping the assumption that the avalanche time scale is negligible compared with the ice build-up time scale. We now only wait a limited number of avalanche time cycles (one avalanche time cycle corresponds to one application of eq. (1) on every $h(x, y) > h_c$) before the next ice addition. This yields an increase of the steepness of the power spectrum due to a larger correlation of sequential ice volume values. If the avalanche time scale is about the same order of magnitude as the ice build-up time scale, we obtain a power-law exponent of -2.4. The spectrum resulting from allowing one avalanche time cycle after ice addition is also shown in fig. 2. It matches quite well the power-law background spectrum of the Quaternary ice volume fluctuations.

These results encourage investigations with a more realistic ice model. We use a two-dimensional (vertically integrated) global model on a 72×72 grid with fixed ice temperature [19]. In its simplest version the model is driven by a snow balance which depends only on the height difference of the ice surface and a fixed latitude dependent snow height limit. This does not yield realistic ice distributions (see fig. 3), which is not important for our investigations, but has the advantage of saving computer time.

We make the following assumptions about the surging mechanism. Besides its normal flow, the ice has a second *fast* flow mode. If the ice is in the fast mode at a certain grid point, it obeys exactly the same flow law as in the normal mode, except that the ice flux is a factor 100 larger. When the ice is in normal mode and the vertically integrated ice flux exceeds a certain critical value q_1, we assume the frictional heating produces basal water, and the ice switches to fast mode. A connection between frictional heating and surging was also assumed in other models [17,18]. When in fast mode the ice flux drops below $q_2 = 10q_1$, the ice switches back to normal mode. The *different* switching points can be justified by a lower heat production – for the same flux – in the lubricated fast mode.

The model is started with no ice. After a phase of ice build up, temporary surging sets in and the model exhibits ice volume fluctuations around a constant mean value, see fig. 3. The variance is only about a few percent and therefore much less than the observed. This indicates that the model lacks some important amplifying feedback mechanism. However, the spectrum of the model ice volume fluctuations obeys a power law with a slope very close to the observed one, see fig. 4.

The confirmity of the two models and the ice volume data suggest that the continental ice sheets are in a self-organized critical state, which provides a very simple explanation for the background power-law spectrum of the Quaternary ice volume fluctuations. This does not contradict the correlation between a portion of the ice volume variations and the earth's orbital parameters if orbital forcing is included in the model. On the contrary, if we subtract the

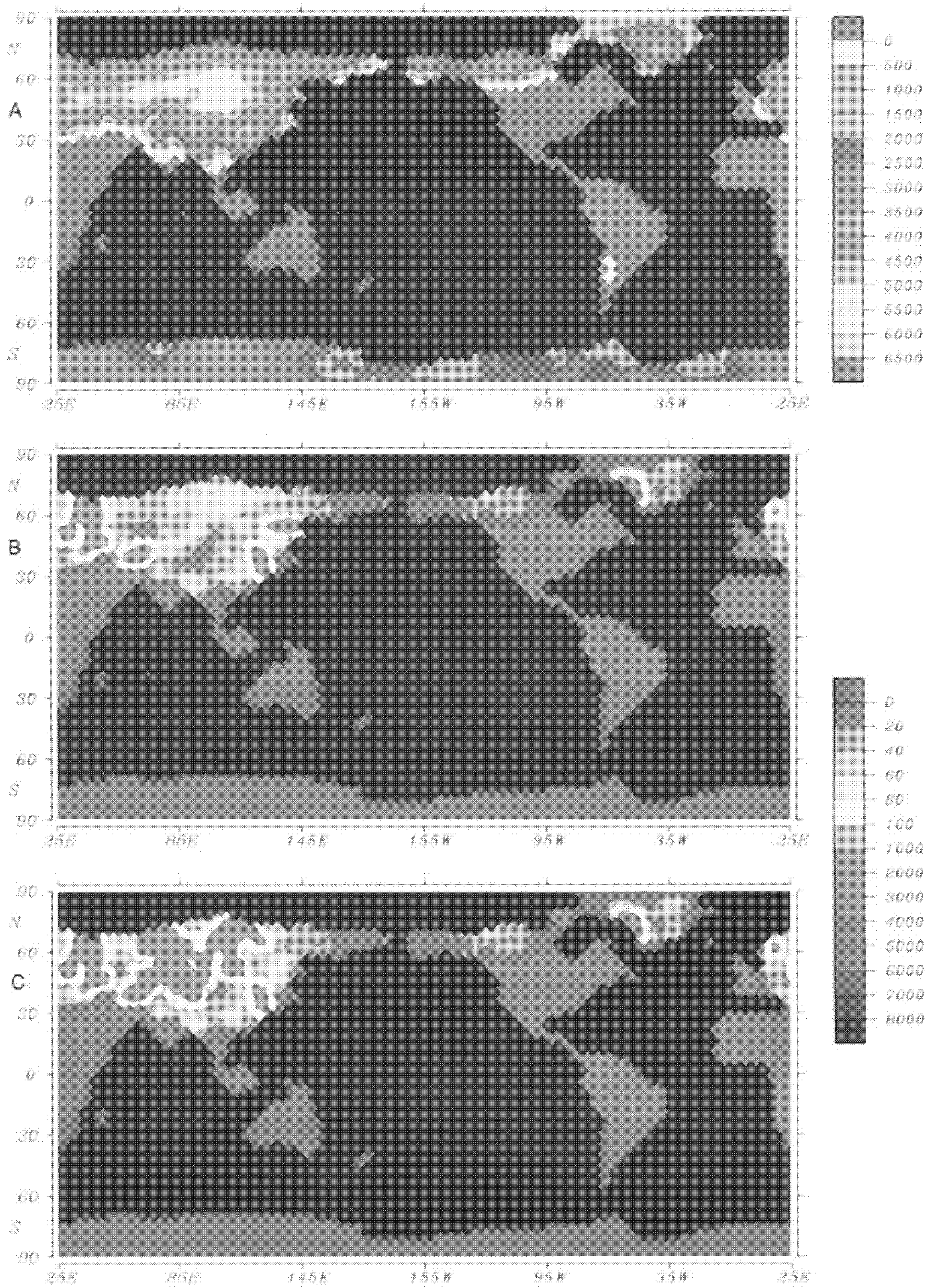

Fig. 3. Snapshot of the global ice thickness distribution (A) and global maps of the ice flux velocity in a relatively quiet period (B) and during a major surge (C). Red indicates grid points in surge mode. The antarctic ice sheet is fixed to save computer time.

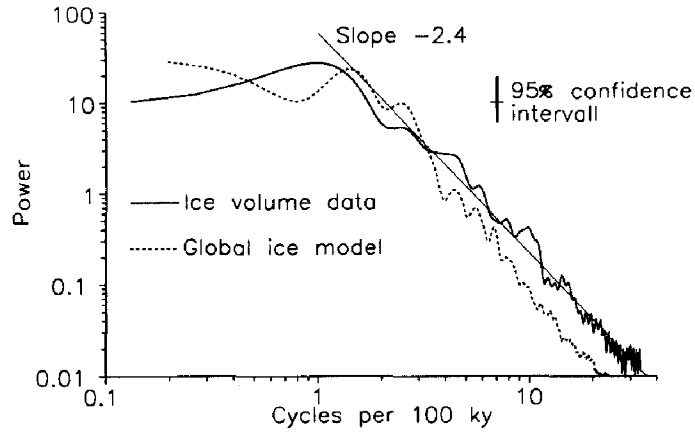

Fig. 4. Comparison of the power spectra of the Quaternary ice volume fluctuations and the volume fluctuations in the global ice model with incorporated surging mechanism.

background power law spectrum due to self-organized criticality, the peak at the 100 ky band is of the same order of magnitude as the peaks at the 19, 23, and 41 ky bands. Therefore it can now easily be explained by some non-linearity in the response to the solar insolation.

References

[1] J. Geikie, The Great Ice Age (Stanford, London, 1894).
[2] J. Croll, Climate and Time (Appleton, New York, 1875).
[3] W. Köppen and A. Wegener, Die Klimate der geologischen Vorzeit (Gebrüder Bornträger, Berlin, 1924).
[4] A.L. Berger, Celestial Mech. 15 (1977) 53.
[5] R.S. Bradley, Quaternary Paleoclimatology (Unwin Hyman, Boston, 1985).
[6] M. Sarnthein, H. Erlenkeuser, R. von Grafenstein and C. Schröder, "Meteor" Forschungsergeb. Reihe C 38 (1984) 9.
[7] J.D. Hays, J. Imbrie and N.J. Shackleton, Science 194 (1976) 1121.
[8] K. Herterich, Max-Planck-Institut für Meteorologie, Report 23 (Hamburg, 1988).
[9] J. Imbrie and J.Z. Imbrie, Science 207 (1980) 943.
[10] D.G. Martinson, N.G. Pisias, J.D. Hays, J. Imbrie, T.C. Moore Jr. and N.J. Shackleton, Quat. Res. 27 (1987) 1.
[11] P. Bak and Ch. Tang, J. Geophys. Res. 94 (1989) 15635.
[12] P. Bak and K. Chen, Physica D 38 (1989) 5.
[13] G.A. Helf, D.H. Solina II, D.T. Keane, W.J. Haag, P.M. Horn and G. Grinstein, Phys. Rev. Lett. 65 (1990) 1120.
[14] P. Bak, Ch. Tang and K. Wiesenfeld, Phys. Rev. A 38 (1988) 364.
[15] P. Bak, Ch. Tang and K. Wiesenfeld, Phys. Rev. Lett. 59 (1987) 381.
[16] W.S.B. Paterson, The Physics of Glaciers (Pergamon, Oxford, 1981).
[17] J. Oerlemans and C.J. van der Veen, Ice Sheets and Climate (Reidel, Dordrecht, 1984).
[18] W.F. Budd and B.J. McInnes, Hydr. Sci. Bull. 24 (1979) 95.
[19] K. Fieg, Diploma Thesis, Max-Planck-Institut für Meteorologie, Hamburg (1992).
[20] J. Imbrie, J.D. Hays, D.G. Martinson, A. McIntyre, A.C. Mix, J.J. Morley, N.G. Pisias, W.L. Prell and N.J. Shackleton, in: Milankovitch and Climate, Part I, A. Berger et al., eds. (Reidel, Dordrecht, 1984) pp. 269–306.